647510 **MSC1MatC101x** **Seat No :________**

M.Sc.(Maths.) Semester - 1 (*CBCS*) Examination
Oct/Nov. -2019 - [NEW COURSE]
Algebra 1(Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q-1 Attempt any seven: **(14)**

(a) Find all generators of $\mathbb{Z}_6$.

(b) Find the order of permutation (13426) in *S*6.

(c) Define Automorphism of groups.

(d) What is the order of any nonidentity element of $\mathbb{Z}_3 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_3$.

(e) Define Sylow p-subgroup.

(f) Define nilpotent ideal and give one example of it.

(g) What are the prime ideals of the ring of integers, $\mathbb{Z}$.

(h) Define Euclidean domain.

(i) Is $x^2 + x + 1$ irreducible over $\mathbb{Z}_2$?

(j) Let *D* be a Euclidean domain with function d. Prove that u is invertible in *D* if and only if d(u) = d(1).

Q-2 Attempt any two: **(14)**

(a) Show that every permutation of a finite set can be written as a cycle or as a product of disjoint cycles.

(b) Prove or disprove: There exists an isomorphism from the group of rational numbers under addition to the group of nonzero rational numbers under multiplication.

(c) State and prove Cayley's theorem.

Q-3 Answer the following. **(14)**

(a) Determine the number of elements of order 5 in $\mathbb{Z}_5 \oplus \mathbb{Z}_5$.

(b) If *H* is a subgroup of a finite group *G* and |*H*| is a power of a prime p, then show that *H* is contained in some Sylow *p*-subgroup of G.

OR

Q-3 Answer the following. **(14)**

(a) Let G and H be finite cyclic groups. Then show that $G \oplus H$ is cyclic if and only if |*G*| and |*H*| are relatively prime.

(b) Determine all groups of order 99.

Q-4 Attempt any two: **(14)**

(a) Let *R* be a commutative ring with unity and let *A* be an ideal of *R*. Then show that *R/A* is an integral domain if and only if *A* is prime ideal.

(b) Find all maximal ideals of $\mathbb{Z}_{36}$.

(c) Prove that the only homomorphisms from the ring of integers $\mathbb{Z}$ to $\mathbb{Z}$ are the identity and zero mappings.

Q-5 Attempt any two: **(14)**

(a) Show that Every Euclidean domain is a principal ideal domain.

(b) Show that *F[x]* is a unique factorization domain where *F* is a field.

(c) State and prove Eisenstein Criterion.

(d) Construct a field of order 25.

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